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Oil Deposits in Diatomites: A New Challenge for Subterranean Mechanics

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Abstract

Because of their size and difficulties with oil recovery, the oil-bearing diatomite formations attract now special attention worldwide. For example, the giant diatomaceous oil fields in California, Lost Hills and Belridge, contain some 10 billion barrels of oil in place. Diatomaceous strata have peculiar geological structure: as a result of the cyclic deposition, the diatomite rocks are layered across width scales ranging from tens of meters to sub-millimeter. The diatomite rock is very fragile and its fracture toughness is low: the inter-layer boundaries are weakly connected and ready to part when the fluid pressure changes. When intact, the diatomite has porosity of 50-70% and is almost impermeable (0.1-1 md). Oil production from the diatomites was always difficult and started only 30 years ago after the introduction of hydrofracturing. The scanning electron microscopy images of the diatomite rock reveal a disordered microstructure with little grain interlocking and cementation. Therefore, fluid flow through the diatomite starts only after changes of the rock microstructure. The hydrofractures are not single vertical cracks, but are complex, multiply connected regions of damaged rock.

The current models of fluid-rock systems, e.g., Refs.,[1,3,19] cannot capture the dramatic rearrangements of the diatomite microstructure caused by fluid withdrawal and injection, and have little predictive capability. In particular, these models cannot capture the intense rock damage during hydrofracturing, followed by the nonequilibrium countercurrent imbibition with the ensuing rock damage and hysteretic effects. To understand and predict reservoir behavior in the diatomite and limit well failures, a new micromechanical approach has been developed.

Introduction

We face nowadays a new period in the development of subterranean mechanics: the science of flow, deformation and fracture in natural rock-fluid systems. Important practical problems with oil and gas recovery, water supply, and more recently with the disposal of nuclear and chemical wastes, are forcing researchers to reconsider and modify the established flow theories that, with relatively minor changes, have dominated earth sciences since the late thirties. These modifications lead to substantially new physical and mathematical formulations, and are required to tackle practical problems with, e.g., oil production from the North Sea chalks and the diatomites in California, liquid nuclear waste seepage at Hanford, WA, or with nuclear waste isolation under the Yucca Mountain project, NV.

In classical approach, mass, momentum and energy conservation laws and their invariance properties alone are insufficient to obtain a closed consistent system of equations of fluid flow in real rocks. To obtain such a system, it is necessary to endow continua with physical properties. This means that a model of each continuum is employed, which is adequate for the classes of fluid and rock motions of interest. Such models, called *constitutive equations*, provide the necessary relationships between the properties of the motion, or the states of the continua, and acting internal forces.

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A constitutive equation becomes a fundamental physical law if the constants entering it are universal. In particular, this means that once determined from experiment, these constants are the same within a wide range of conditions. Such constants are Young's modulus and Poisson's ratio for elastic isotropic rocks, and the viscosity coefficient for Newtonian viscous fluids.

For complex rocks, e.g., chalks or diatomites, the researchers have tried to extend the classical models by including more constants, as in the five-constant Drucker-Prager model[11] of elastic and plastic rock deformation. The increasing complexity of determining the new constants and the loss of their clear physical sense are not the worst outcomes: the biggest limitation of the Drucker-Prager and similar models of diatomite and chalk is that their constants cease to be universal, i.e., their range of universality may be so narrow as to eliminate the model's predictive abilities.

A mere inspection of the numerous well failures in the South Belridge Diatomite field[27] is sufficient to disclose the universality of deformation and rupture of the reservoir rock and overburden under the conditions prevailing there, an example is shown in **Fig. 1**. Hence, it is necessary to quantify the effects of the stress-strain state which produces this deformation and fractures. The reservoir conditions at South Belridge are characterized by high and nonuniform gradients of pore pressure (and effective stress) near the injectors and the producers. What properties of the diatomaceous rock are conducive to its rupture and flow under the existing and imposed shear stresses? The low diatomite rock strength (fracture toughness) and fine layering seem to be the two most important properties causing its easy damage and flow.

To describe the fluid flow in fragile, prone to damage, and practically impermeable in pristine state rocks, we use an alternative approach that appeared in the last four decades.[2,4,5,7,14] In this approach, the microstructural properties of the material, directly observed or computed from the fundamental conservation laws, are explicitly introduced as a part of the model. In particular, the damage parameter understood as the average fraction of broken bonds in the rock is considered explicitly. The macroscopic fluid flow equations and the equations governing the kinetics of microstructural rock transformation are solved simultaneously. This micromechanical approach is the essence of our work on flow and damage of the fragile reservoir rocks.

Properties of Diatomite Rocks

Rock Microstructure. The microstructures of the North Sea Chalks or the California diatomites are fundamentally different from, say, sandstones. **Fig. 2** shows the scanning electron microscope (SEM) image of sandstone magnified 150 times. The pores are the dark regions surrounded by about six grains similar to beach sand in size. The interlocking grains form a network of strong beams that together protect the pore space from a collapse caused by

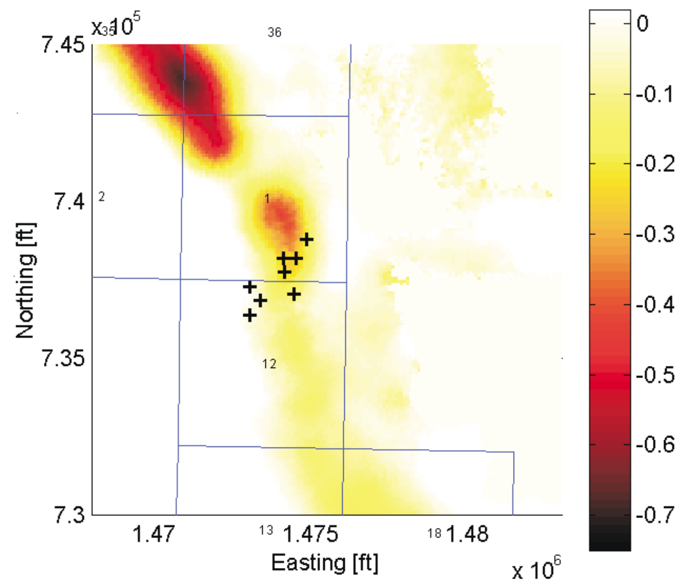

Figure 1: The wells that failed in Sections 11 and 12 of the South Belridge Diatomite in the year 2000 were located outside the nearest large subsidence bowl. Surface subsidence is in mm/day. The InSAR interferogram was processed by Dr. Eric Fielding of JPL/NASA.

increasing effective stress.[32]

The SEM image of a North Sea chalk in **Fig. 3** is magnified 10000x. The chalk grains are of the size of very fine dust. The pore walls are made of hundreds of coccoliths that form a poorly cemented mosaic. These coccolith mosaics are mechanically weak and collapse with the changing pore pressure. The chalk porosity is 40-50%, and it is practically impermeable (0.1-5 md).

The low (600x) and high (5000x) SEM images of the outcrop diatomite, **Fig. 4**, reveal a disordered microstructure with little interlocking and cementation. When intact, the diatomite has overall porosity of 50-70% and like chalk it is practically impermeable (0.1-1 md).

To make any appreciable fluid flow possible, the chalk and diatomite microstructures must be *rearranged*. The microstructural changes can be caused by changes of effective stress leading to the collapse of pore walls. As a result, a network of microcracks is created, which radically changes the mechanical and flow properties of the two rocks. It then follows that both water injection and oil withdrawal cause a significant and irreversible changes of the microstructures of both rocks. The disintegration of chalk and diatomite implies increased rate of rock compaction during waterflood.

As a result of the cyclic depositional environment,[31] the diatomite rocks are layered across width scales ranging from tens of meters to sub-millimeter. The inter-layer boundaries are weakly connected and ready to part when

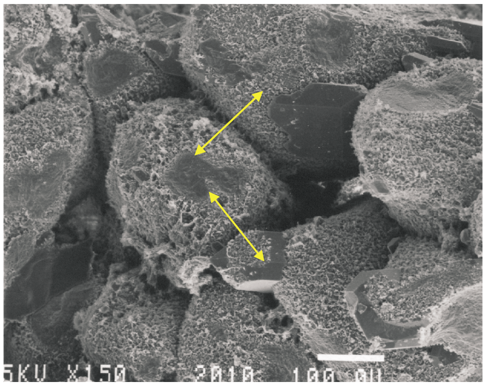

Figure 2: The inter-locking grains of a sandstone form strong "support beams" (SEM photo by K/T GeoServices, Inc.).

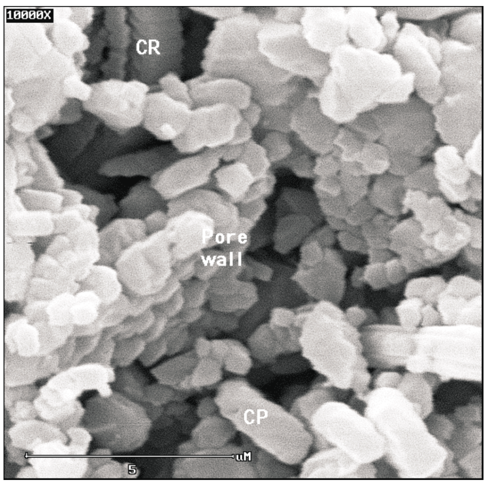

Figure 3: Chalk coccoliths make mechanically weak "mosaic" pore walls (SEM photo by Jess Milter[22]).

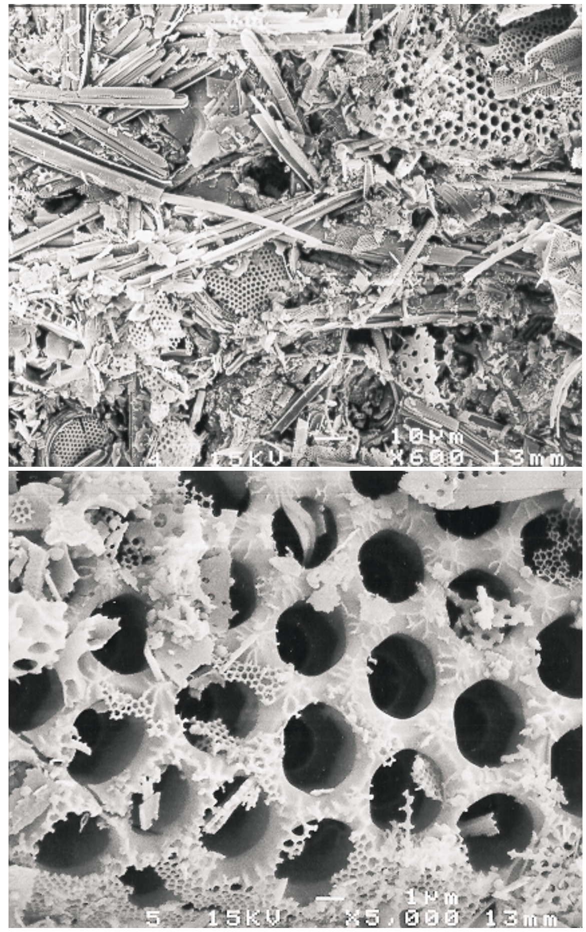

Figure 4: SEM microphotographs of the diatomite rock show its complex and fragile microstructure. When compared with chalk, the diatomite has a higher porosity and is weaker mechanically (SEM photos taken at Berkeley's Civil and Environmental Engineering Electron Microscopy Lab).

the fluid pressure changes. When a Lost Hills diatomite core dries somewhat, and shrinks, macroscopic horizontal fractures appear at an almost uniform spacing of about 1 cm. Large-scale faults and fracture systems are also known to exist[23] in the diatomite fields. The large fracture systems will probably increase their size and connectivity when the pore pressure changes. For example, some of the current infill wells in the Lost Hills waterflood patterns free-flow prior to hydrofracturing, and may produce 10 barrels of oil per day. At the beginning of primary production in Lost Hills, oil production from a non-fractured well was practically zero. This observation allows us to assume that the new vertical wells intersect a predominantly horizontal system of fractures or high permeability layers, which are (1) initially filled with oil, and (2) pressurized by water from the adjacent injectors. As the micro-cracks created during field operations increase in number and connectivity, they will connect to the macro-fracture system. Therefore, production and injection in the diatomite will result in the interactions of fractures at many scales.

An aqueous tracer test in Section 32 of Lost Hills, revealed[28] that the upper injection hydrofracture linked with four producers through the flow channels or "tubes," probably in a single highly-permeable layer, that are about 100 times more permeable than the rock matrix, and have effective cross sections of the order 10-100 cm^2. The results of the tracer tests in near by CO_2 pilot patterns were similar.

Fluid-Rock Interface Microstructure. In an oil and gas reservoir, the microstructure of fluid-fluid and fluid-solid interfaces is as important as that of the rock. For example, it is important to know in which pores the oil resides, and which pores are filled with water. One would expect the oil to be present in the largest pores, microcracks and fractures, and water to reside in the micropores and intraparticle pores. From this point of view, the undamaged

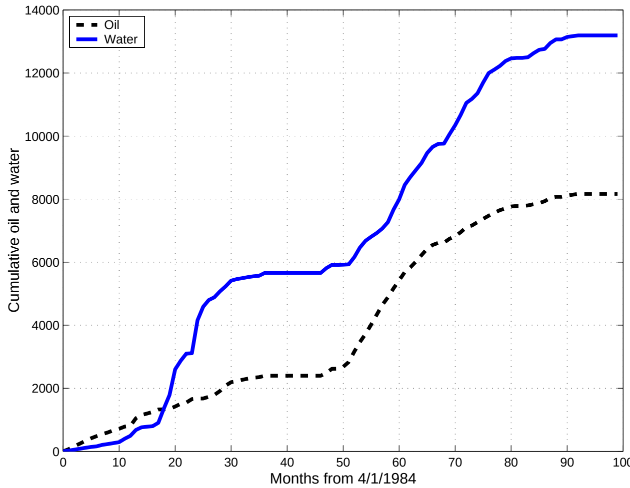

Figure 5: Initially low water production during primary in a Lost Hills well dramatically increases after several months. There was no water injection until 1995, and there is no aquifer; therefore, the produced water came from the crushed microporosity.

chalk and diatomite will slowly produce oil with little water. As the reservoir pressure decreases around the well or "hydrofracture," the rock collapses, and the water trapped in the smallest pores is released, **Fig. 5**. The suddenly abundant water production may also mean that the natural fractures are predominantly oil-wet, possibly because of a substantial asphaltene[17] content[i] in the crude oil.

The issues of pore occupancy, wettability, and pore-level displacement mechanisms will be addressed through a separate research program that will involve the freeze-SEM images of the fluid microstructures, the synchrotron micro-focused computed tomography imaging of the rock microstructure, and the pore-network modelling of drainage and imbibition with two and three fluid phases and wettability alteration.

Hydrofractures. All wells in the diatomite are hydrofractured, and the vertical "fractures" are thought[37] to have tip-to-tip lengths of the order of 100-300 ft and heights of 50-300 ft. In reality, these "hydrofractures" are complex volumes of pulverized or liquified soft rock with complex connectivity and geometry.[12,15,16] The damaged rock volume around a hydrofracture may give an illusion of a vertical crack, but it is not. There are several reasons for this behavior:

1. A sudden increase of the pore pressure in the water-fractured well cannot be diffused away by the almost impermeable diatomite. The weak rock is damaged and close to the well becomes liquified, while its microstructure collapses. Description of this phenomenon with a single Young's modulus and Poisson's ratio makes little sense. The Poisson's ratios measured in the diatomites vary significantly, approaching one at times.

2. The layer boundaries are natural planes of weakness, and the diatomite damage grows outwards as horizontal irregular fingers, **Fig. 6**. The growth of some of these fingers was captured as microseismic events,[12,16,25] 25-30 ft away from the hydrofracture "plane".

3. There is no natural upper confining layer for a "vertical hydrofracture" in the diatomite. As water is injected, it continuously liquifies the rock in the uppermost parts of the damaged rock zones. The vertically extending damage eventually turns genuinely horizontal because the overburden stress becomes the least principal stress. Once this happens, the vertical damage extension stops. In particular, the "hydrofractures" may grow out of the diatomite and turn horizontal in the overlying unconformity of much higher permeability. At higher injection pressures, the horizontal fracture may be created at some distance above the injection interval.[16,25] From then on, most of the injected water ends up in the horizontal fractures and/or the overlying sands.

4. The lower hydrofractures are generated in the harder, more brittle opal CT[ii], to which the amorphous organic silica converts with depth and temperature.

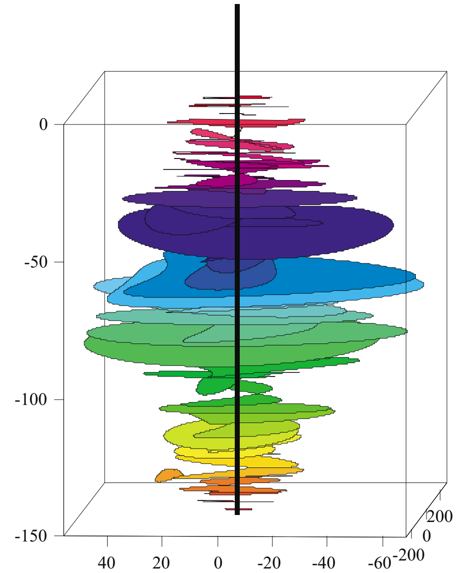

Figure 6: Cartoon of the view down a shallow hydrofracture in the diatomite. The fracture does not resemble a single narrow crack in hard rock. Instead there are many complex horizontal fingers of damaged diatomite, idealized here as ovals, in distinct layers.

[i] Private communication from Dr. E. Dezabala, ChevronTexaco.

[ii] Opal CT is a crystalline variety of silica characterized by a poorly ordered fabric, often an intermediate phase in quartz chert formation.

These hydrofractures may resemble multiple vertical cracks.[15,16]

5. The outer surface of cement around well casings may be another place where the weak diatomite fails. This mode of failure would result in water injection into the overlying sands without any "fracture extension".

Waterflood patterns in the diatomite are usually configured as staggered line drives, but these patterns do not always follow the direction of maximum horizontal in-situ stress. Depending on the fracture orientation and flow direction, areal sweep by water may vary greatly.[8] Multiscale layering in the diatomite[23,31] and rock damage[6,10] result in strong anisotropy and nonuniform vertical and areal sweep by the injected water.

An Outline of the Mathematical Model of Waterflood in the Diatomites

In this section we present a principal scheme of the fluid flow in a diatomaceous rock. This problem, when properly elaborated, should lead to an essentially new statement for subterranean fluid mechanics, which did not appear previously. We give here only an outline of the approach as we see it now. The detailed implementation of the new approach requires substantial amount of numerical, laboratory and field experiments. However, even in its present state the model can be helpful for qualitative estimates and predictions.

Basic Classical Model. As a reference classical mathematical model we use the simple model of an elastic drive in the oil deposits of platform[iii] type, **Fig. 7**.

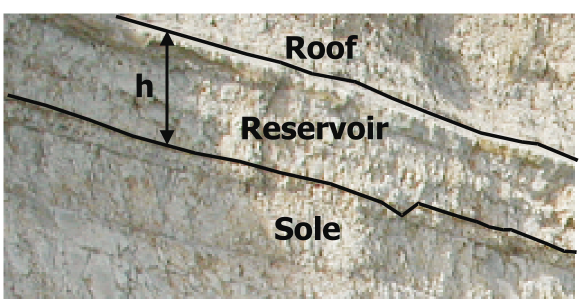

Figure 7: An isotropic oil-bearing stratum, i.e., a clearly discernible layer of sedimentary rock. Above and below the reservoir rock are the roof and sole, which may be permeable. In the diatomite, the stratum may correspond to a single hydrofractured interval, and the sole and roof may correspond to the unfractured "blank" zones of poor-quality pay that separate the hydrofractured intervals.

[iii]The platform type reservoirs are those rock formations whose thickness is much less than areal extent. Examples are the Upper Guadalupian platform sandstones, Romashkino, etc. Because the diatomite deposits are very thick, here we consider just one major cycle.[31]

If the height, h, of the stratum is much less than its lateral extent, L, the basic equation for the fluid pressure, p, in predominantly horizontal flow, see, e.g., Dake[9] or Ref.,[3] can be averaged with respect to the height yielding:

$$\phi h(c + c_f)\frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{kh}{\mu}\nabla p\right) + q, \qquad (1)$$

where t is the time; ∇ is the standard notation for the differential operator (gradient and/or divergence); ϕ is the vertically-averaged rock porosity; c and c_f are the vertically-averaged compressibility coefficients of the fluid and the rock porosity; k is the vertically-averaged permeability coefficient; and μ is the fluid dynamic viscosity.

We allow for the vertically-averaged permeability and porosity to be functions of the horizontal location, x and y. We do assume, however, that the vertically-averaged permeability is isotropic.

In most reservoirs, the coefficient of productivity, kh/μ can be determined by well-testing procedures.[21,35] In the diatomite, obtaining the formation productivity factor from well testing has been a daunting task, because of the exceedingly low rock permeability. Just a couple of such tests have ever been performed. The permeability of the reservoir rock surrounding the hydrofractures is usually estimated[38] from the slope of the cumulative oil vs. $\sqrt{t}$ lines, i.e., from the solution of the pressure diffusion equation in linear transient flow. This method has proved to be quite reliable; other methods also exist.

The total inflow rate, $q = q_r + q_s$ is the net fluid inflow through the roof, q_r, and sole, q_s, of the stratum. Equation (1), used for the reservoir elastic drive, was obtained by Theis[36] and Jacob,[13] and later independently by Schelkachev,[29,30] who took into account the rock compressibility and so was able to explain the effect of supercompressibility[iv]. The inflow q usually is not taken into account because the roof and sole are assumed to be impermeable. It is important that this may not be the case for the diatomite layers.

As the boundary conditions for equation (1), the pressure or flux through the rows of wells (or a single well in well-testing) are prescribed.

A well-known complication of this basic formulation is related to the separate consideration of water and oil flow. In this case, the phase permeabilities enter, as well as capillary pressure, and an equation for the water (or oil) saturation should be added. Here we will not consider this complication, although there are no principal difficulties in accounting for this effect.

Damage. We summarize the basic properties of the diatomite rock in the pristine state, as it is in geological for-

[iv]Theis did not consider rock compressibility and, to match the observed aquifer compressibilities, the "supercompressibility" of water with air bubbles was invented. Schelkachev showed that by taking into account the rock compressibility, the fluid supercompressibility need not be invoked.

mations before the beginning of the oil field development considered in the previous section:

1. The rock is highly porous.

2. The rock is microlayered and its fracture toughness is low.

3. The rock is practically impermeable.

Using the terminology of fracture mechanics,[18] we can say that the fluid flow in the diatomites begins only when *damage* appears in the rock. There exist various quantitative definitions of damage. Most rigorous definition is as follows:

Definition: Damage ω is the relative amount of broken bonds between the elements of the material (here rock) microstructure.

The value of damage lies between $\omega = 0$ (pristine rock) and $\omega = 1$ (pulverized or liquified rock). The quantitative measurement of damage ω *in situ* is a special and difficult problem in damage mechanics; nevertheless, this concept has worked in practical predictions of the life-time of structures made of steel, fiber-reinforced concrete, etc. There were attempts to introduce damage as a tensor quantity - we will not do it here, trying to simplify the basic concept as much as possible, but such complication can be incorporated in the future.

Principles of Mathematical Modelling of Fluid Flow in Diatomites. Our proposal is to use for flow in the diatomaceous rocks the same basic equation (1) with one important distinction: the rock permeability k is no more a fixed quantity for a given point of stratum, but it becomes a function of the rock damage: $k = k(\omega)$. Thus, equation (1) takes the form:

$$\phi h(c + c_f)\frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{k(\omega)h}{\mu}\nabla p\right) + q \tag{2}$$

so that an equation for the evolution of damage, ω, should be added to obtain a closed system of equations.

The determination of the absolute permeability function, $k(\omega)$, is a special problem in rock mechanics that will be addressed in a later paper. The permeability, k, is a fast growing function of damage, ω; by definitition, $k(\omega = 0) = 0$. The permeability of a damaged rock can also depend on the fluid pressure. In subterranean mechanics there is experience in working with pressure-dependent rock permeability[v], so such a dependence can be easily implemented, but in the present work we do not consider it. Note that the rock porosity is a much weaker function of damage and we leave it as a function of the fluid pressure alone. Field observations suggest[24,27,28] that in the diatomites the initial porosity changes are small (from, say, 0.5 to 0.45), but the permeability changes can be very large (from 0.5 md to 50 md).

In classical damage mechanics in one spatial dimension an equation for damage kinetics is assumed to be[14]

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = F(\omega, \sigma^{(t)}) \tag{3}$$

where F is a function of current damage, ω, and the true stress, $\sigma^{(t)}$, acting on the undamaged surface portion of the rock; several versions of this function have been proposed.

For the diatomite rocks, the high microinhomogeneity of the fracture process, i.e., of damage accumulation is a characteristic property (cf. the discussion above). In Refs.[2,4] it was demonstrated that the microinhomogeneity leads to a special effect: *the diffusion of damage*, so that the basic equation of the rock damage takes the form:

$$\frac{\partial\omega}{\partial t} = \nabla \cdot (\mathcal{D}(\omega, I_1)\nabla\omega) + \mathcal{F}(\omega, I_1) \tag{4}$$

Here $I_1 = \sigma_x + \sigma_y + \sigma_z$ is the first invariant of the average effective stress tensor in the rock, and σ_i, $i = x, y, z$, are the respective principal effective normal stresses. For simplicity, we again do not take into account the possible tensor character of the *damage diffusivity* $\mathcal{D}(\omega, I_1)$. The *damage generation function* $\mathcal{F}$ must also be specified for the diatomite; usually it is a very strong function of stress in the rock.

The quantity

$$\sigma = \frac{1}{3}I_1 \tag{5}$$

is known as the mean or hydrostatic stress. Due to quasistatic equilibrium of the rock-fluid system, the sum of fluid pressure p and σ is approximately constant at a given depth,[3] and so we come to the final equation for the rock damage evolution:

$$\frac{\partial\omega}{\partial t} = \nabla \cdot (\mathcal{D}(\omega, p)\nabla\omega) + \mathcal{F}(\omega, p) \tag{6}$$

Note that for simplicity we use the same notations for $\mathcal{D}$ and $\mathcal{F}$ as in Eq. (4). Equations (2) and (6) form a system of two coupled partial differential equations for the pressure and damage, under the assumption that the functions entering these equations are known. We repeat that the determination of these functions will require subsequent computational, laboratory and field experiments; however, we know now enough to outline the structure of these functions and design the necessary experiments.

Boundary Conditions. We can assume that the rock is fully pulverized or liquified at the injection wells after the hydrofracture, so that at the well, condition $\omega = 1$ should be satisfied. The most important feature of the problem is

[v] For example, pressure-dependent permeability is a standard feature of most existing reservoir simulators.

the following. Each injection well occupies a certain volume (area in 2D) of the stratum. Due to water injection, the damaged area, Ω, **Fig. 8**, occupies some part of the stratum, and in the remaining part the rock remains undamaged, so that $\omega = 0$, and there is no flow. Therefore, at the "material" boundary,[20] Γ, of the damaged region, the following conditions should be fulfilled:

$$\omega = 0, \quad \mathcal{D}\frac{\partial \omega}{\partial n} = 0 \quad \text{on } \Gamma \tag{7}$$

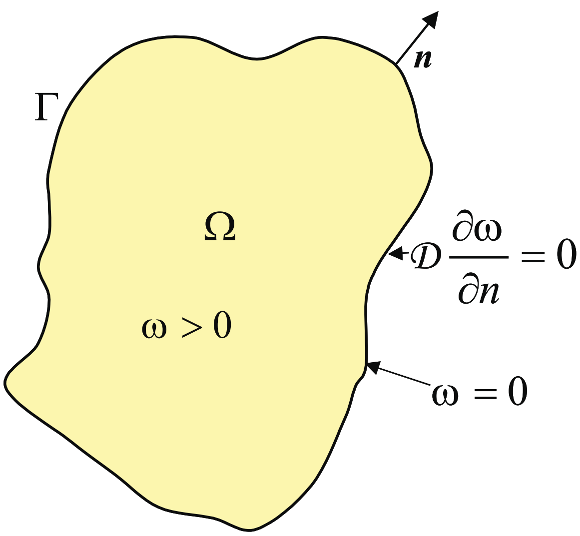

Figure 8: The time-dependent area, Ω of damaged rock around an injection well bounded by a time-dependent free-boundary Γ. The Γ curve is "material", i.e., both $\omega = 0$ and $\mathcal{D}\partial\omega/\partial n = 0$ along Γ; $\boldsymbol{n}$ is the unit vector normal to Γ.

It is known in the theory of differential equations that if both conditions (7) are prescribed at a given *known* curve Γ, the problem has no solution. The special property of our problem is that the curve Γ is a *free boundary*, i.e., it is *unknown* and should be found as part of the solution. In this formulation, the fluid-flow/rock-damage problem is reduced to the solution of the free boundary problem for the system of equations (2) and (6). The mathematical techniques of constructing the solutions of such a problem are generally known and can be applied here.

Discussion

Diatomite rocks and other rocks of similar structure form the strata of oil fields of considerable value worldwide, containing tens of billions of barrels of oil. At the same time, the mathematical modelling of such deposits using classical approach is inappropriate due to the reasons listed above.

This paper contains an outline of the appropriate mathematical model of water injection: a free boundary problem for a coupled system of partial differential parabolic equations for the evolution of the fluid pressure, p, and the rock damage, ω. These equations require constitutive relationships for the rock permeability, k, as a function of ω, and the damage diffusivity and generation, $\mathcal{D}$ and $\mathcal{F}$, respectively, as functions of ω and p. This system of equations allows one to come to several qualitative conclusions.

Return, for example to Fig. 1. The boundary of subsidence is clearly seen. It is instructive that all failed wells lie outside of the region of strong subsidence. This means that the damaged region of the reservoir, where ω is positive, lies outside the clearly visible subsidence region and, therefore the free boundary of the damaged region is also outside of the clearly visible subsidence bowl. The rock strength in the damaged region is reduced, the damaged rock creeps, and the wells fail.

The numerical solution of the presented model will allow one to predict growth of the damaged rock region as a function of time. It will also allow one to recommend an optimal strategy of *damage management*.

Conclusions

Summing up, we can say that the current understanding of mechanisms of oil production in primary and waterflood is insufficient for the diatomites, and also for the chalks. Consequently there is little if any capability to predict the ultimate oil recovery and the rate of well loss caused by the nonuniform subsidence. Given a reasonable injection policy, runaway damage of the rock can be reduced with the new injection controllers,[26,33,34] but the injection profiles and areal sweep by water are currently uncontrollable and may adversely affect the ultimate oil recovery.

Our motivation is to help produce oil from the practically impermeable California diatomites by improved oil recovery techniques that may involve water or gas injection. Because of the unusual nature of the diatomite fields, we have to reconsider the currently accepted standards of oilfield development.

Acknowledgements

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Nomenclature

h	=	reservoir thickness, L
c	=	fluid compressibility, $\mathsf{L}^2\mathsf{F}^{-1}$
c_f	=	porosity compressibility, $\mathsf{L}^2\mathsf{F}^{-1}$
$\mathcal{D}$	=	coefficient of damage diffusivity, $\mathsf{L}^2\mathsf{T}^{-1}$
F	=	damage generation function in mechanics, T^{-1}
$\mathcal{F}$	=	damage generation function in rock, T^{-1}
I_1	=	first invariant of effective stress tensor, FL^{-2}
k	=	absolute permeability, L^2
$\boldsymbol{n}$	=	unit outward normal vector
p	=	fluid pressure, FL^{-2}
q	=	inflow rate, LT^{-1}
t	=	time, T
x, y, z	=	Cartesian coordinates, L
μ	=	coefficient of fluid viscosity, $\mathsf{FL}^{-2}\mathsf{T}$
ϕ	=	porosity
σ	=	hydrostatic stress, FL^{-2}
$\sigma^{(t)}$	=	true stress, FL^{-2}
∇	=	differential operator, L^{-1}
Γ	=	boundary of damaged zone
ω	=	damage
Ω	=	volume (area in 2D), L^2

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